Modelling Inflation Using Markov Switching Models:
Case of Poland, 1992 – 2005

Abstract

We investigate inflation in Poland in the period of economic transition by examining the potential application of Markov Switching Models to model the inflation generating process in Poland. The time horizon of analysis was limited to the period between March 1992 and October 2005 defined as the process of disinflation, i.e. the process of continued decrease in inflation rates following the economic transition period in early 1990s which was accompanied by a high level of inflation.

According to the Ball-Friedman hypothesis, variation of inflation during periods of high inflation can be unstable. Indeed, the results show that non-linear models significantly improve the description of inflation generating process in Poland. Apart from univariate Markov Models, we also use a model that incorporates inflation expectations measured by Future Inflation Indicator (FII). We find that the model, where lagged values of FII are included as exogenous variables is significantly better in modelling inflation than simple univariate Markov Model.

Key Words: Markov Switching Models, Inflation modelling, Leading Indicators
1. Introduction

At the beginning of the economic transition in Poland in early 1990s monthly inflation rate reached 50%. Although the conventional threshold of hyperinflation, i.e. 50% month-on-month, has been exceeded only for one month, Polish economy was on the edge of hyperinflation for the following few months. Moreover, high inflation (above 10% yearly) has hampered Polish economy for the following ten years.

Friedman (1977) suggested that higher average inflation should result in more inflation uncertainty because it distorts relative prices and introduces additional risk to nominal contracts. On the basis of Barro-Gordon framework Cukierman and Meltzer (1986) showed in a formal way that higher inflation should be accompanied by higher inflation uncertainty. However, in their model the causality runs in the opposite direction than the one that Friedman pointed out - from uncertainty to inflation. In their model, thanks to discretion and asymmetric information, monetary authorities can increase product by increasing inflation unexpectedly. Nevertheless, Friedman's idea was formally proven by Ball (1992). In his model two types of policymakers with different preferences randomly alternate the power. Higher inflation uncertainty results from higher inflation because economic agents do not know when two types of policymakers swap the power.

Irrespective of which of these hypotheses is true - Ball-Friedman’s or Cukierman-Meltzer’s one - modelling inflation in Poland during the period of transition could be a tough task. In a simple linear regression residuals should be homoskedastic, and higher inflation in the beginning of 1990s was indeed connected with higher inflation volatility.

GARCH-type models could be used both to model higher variance of higher inflation and to test one of these hypotheses. Using data from G7 countries Grier and Perry (1998) showed that higher inflation uncertainty leads to an increase in inflation. The same conclusion comes from Baillie et al. (1996). On the other hand Brunner and Hess (1993) and Grier and Perry (2000) provide also evidence for an opposite causal link. Recently other than classical GARCH approaches have been also used to analyse the problem. Chin-Chuan Yeh (2006) for instance used quantile regression. Caporale and Kontonikas (2009) showed that GARCH models with conditional mean and conditional variance could be adapted to distinguish between short- and long-term inflation uncertainties. On the other hand, in Markov-type models, regimes were associated with different dynamics of the process that generates inflation.

The model proposed by Ball (1992) inspired us to give a new interpretation of the two regimes arising in the analysis of inflation process using Markov Switching (MS)
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models (see e.g. Hamilton 1989 and 1994). The regimes could be associated with two types of policymakers or two types of their preferences as in the Ball model. We think it is an additional theoretical argument in favour of using MS models in modelling inflation.

Structural models of inflation usually consist of a system of equations that depends on macroeconomic policy and expectations of economic agents. A good example of such models is the model used in the National Bank of Poland to forecast inflation (Kłos et al. 2004). The most significant difference between structural and MS models might be observed in impulse propagation mechanism. Structural models allow only for transmission of impulses but do not allow for any changes in the functioning of the economy. However, different regimes might shift inflation expectations and MS provides a framework to model inflation process in an economy subject to such structural changes. This view is also supported by findings by Ricketts and Rose (1995), and Simon (1996), who argue in favour of MS models and underline a possibility to intuitively and elastically model structural changes.

In our approach we use Markov switching models to describe the inflation process in Poland during the transition period and also the process that generates the Future Inflation Indicator (FII) — a composite leading index of inflation in Poland. We justify for the inclusion of the FII by making an observation that inflation can be forecasted using other variables that are leading with respect to their level or dynamics. At the same time such a forecast may be a proxy for the baseline inflation expectations, i.e. inflation expectations conditional on information currently available. The lag of inflation with respect to the business cycle dynamics observed in the USA usually amounts to seven months (Niemira and Klein 1994).

We expect to find the period for which relation between the FII and inflation was stable and for which a simple regression could be used to estimate inflation. We proceed in this way because higher variance of inflation at the beginning of 1990s is not associated with higher variance of the FII. However, in that period the FII is on its highest levels which suggests that the FII could be used to model variance of inflation and, on the basis of Ball-Friedman hypothesis, indirectly model the level of inflation. We also test, whether MS models can be applied to the modelling of time series in a prolonged period of economic transition. The models enable us to cut the time series into pieces in which the relation between the two processes is stable and can be described by linear regressions. We use the procedure to evaluate each sub-process

57 Future Inflation Index (FII), developed by Maria Drozdowicz-Biec, published by BIEC.
separately. In addition, if data generating process undergoes structural changes then the procedure might significantly improve forecasting characteristic of the model.

The remaining part of the paper is organized as follows: In Section 2, we discuss characteristics of inflation and the volatility of inflation in Poland. Section 3 focuses to a brief description of the MS models applied in the analysis. Then, in Section 4 we propose the results of our statistical analysis and show how the FII can be incorporated into these considerations for the whole period between 1992 and 2005. As the analysis of processes generating the FII and inflation shows that they might be considered stable in the period after August 1999, we employ this indicator to increase the ability of the MS model to describe the process of inflation in Poland after 1999.

2. Inflation in Poland between 1992 and 2005

Figure 1 depicts yearly growth rate of core inflation in Poland. Already on the basis of a graphical analysis it can be noticed that neither mean value of inflation nor its variance can be considered constant.

![Figure 1. Core inflation in Poland between March 1992 and October 2005 (annual growth rates %). Source: National Bank of Poland.](image)

In the period after 1989 one could observe in Poland a process of constant disinflation, i.e. a period of decreasing inflation rate. Although disinflation is quite common in developed economies the transition process of the economy makes it unique and difficult to analyse. Henry and Shields (2004) conducted a detailed research
for G–7 countries and noticed that the average inflation in these countries decreased from 10% in 1974–1983 to the level of 4% after 1996. The Polish economy however was characterized in that period by the scale of this phenomenon, its absolute uniqueness and relatively short period in which it took place. As Golinelli and Orsi (2002) noticed modelling of economies in transition is even more complicated because:

a) The period in which prices were determined by the market is too short.

b) The structural changes associated with transition of these economies significantly distorted relations between inflation, money supply, wages and exchange rates.

It is a common practice that the transition period and the period after the transition are modelled separately. Additionally from (b) it can be concluded that it would be pointless to look for a stable relation between the FII and inflation as all the relations between economic series in the Polish economy were probably significantly distorted. During the period of economic transition there was a continuous disinflation while the variance of inflation was very unstable. There were time intervals when it was relatively high and relatively low. Figure 2 presents the changes in moving inflation variance in the analyzed period.

![Figure 2. Moving variance of inflation (seven months moving window) between March 1993 and October 2005. Source: Own calculations.](image)

58 The moving variance was calculated similarly to moving average. There has been calculated, in each period t, the variance from the subsample t − 3, . . . , t + 3.
It might be noticed that at the beginning of 1994 there was a period of declining inflation and six quarters later inflation was subjected to large fluctuations. It might lead to a conclusion that the transition period itself is not homogeneous and inflation depended on different regimes of the monetary policy. This is a motivation to apply some non-linear modelling methods. In this paper we focus on the MS models which are presented in the next section.

3. Markov Switching Models

3.1. Model formula, modelling elasticity

The main idea behind Markov Switching (MS) modelling is that observed system switches between a finite number of discrete states according to an unobserved process. In the context of econometric modelling we can think of discrete hidden states as different fiscal/monetary policy regimes or different levels of economic activity (see Hamilton 1989, Hamilton 1994, Krolzig 1997:2).

Consider the following model:

\[ y_t = \beta_s \cdot z_t + \tilde{\eta}(s_t) \]  

where \( \tilde{\eta}(s_t) \sim N(0, \sigma_s^2) \) and \( z_t \) is a vector of all exogenous variables of the model and lags of \( y_t \). Vector \( \beta_s \) is a vector of regression parameters which depends only on regimes \( s_t \in \{1, 2, \ldots, N\} \). In the MS setting we extend the data generating process by an unobserved process \( (s_t)_{t=0}^\infty \) being an irreducible and ergodic Markov chain. In that case, the process is completely described by its initial distribution \( N \times 1 \) vector and transition probabilities contained in the \( N \times N \) transition matrix \( P = [P_{ij}] \), where \( P_{ij} \) is a probability of changing a state from i to j.

It is convenient to differentiate the models which involve switching in the mean of the process (Markov Switching in Mean, MSM) from those with switching in intercept (Markov Switching in Intercept, MSI). In this notation we follow Krolzig (1997:1). In the case of a binary Markov chain we can formulate, for example, the following MSM model:

\[ (y_t - \mu_s) = \rho_1(y_{t-1} - \mu_{s-1}) + \rho_2(y_{t-2} - \mu_{s-2}) + \tilde{\eta}(s_t) \]  

Here we assume that the process is piecewise linear autoregressive of order two. The MSI case formulates as follows:
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While considering linear models (i.e. \( N = 1 \)) MSM and MSI formulations are equivalent but they are not if \( N > 1 \). Krolzig (1997:1) ran an in-depth analysis of dynamical features of these two processes. He showed that the main difference between MSM and MSI models is that after a shift in regime, the transition to the new mean (conditional expected value) is smooth in an MSI case and rapid for the MSM model.

3.2. Specification and inference

The procedure of constructing a MS model is often simplified and in practice we will assume that the number of hidden states is known and a set of explanatory variables of the model is given – we consider only \( N = 2 \) and \( N = 3 \).

In our analysis we focus on dynamics of the hidden process. The most common way of presenting this dynamics is by plotting a graph of smoothed probabilities. Let \( Y_t \) be all the information about the process and \( \Theta \) be the vector of parameters of the model.

Then, for example, if we plot smoothed probabilities of the form \( p(s_t = 1 | Y_T; \Theta) \). Then, by convention, values above 0.5 mean that in period \( t \) the system was with the highest probability in the first regime.

It is known result from the Markov chain theory that the expected time of staying in state \( i \) is \( \tau_i = \frac{1}{1 - p_{ii}} \). In the context of econometric modelling, \( \tau_i \) means the expected duration time of the \( i \)th regime, where units are the frequencies of time series under analysis. While modelling the binary MS model for quarterly data, we want \( \tau_i \) to be larger then 2, what allows us to use the word ”phases” for different regimes. It corresponds for \( p_{ii} \) to be larger than 0.5. When we analyze monthly data, by similar argument, we want \( p_{ii} \) to be larger than 0.75, which assures that \( \tau_i > 4 \) months. In our applications we don’t make such a restriction in case of three state models because it often happens that one state corresponds to the regime of high growth (or decline) of the process, which is essentially short-lasting in our time series.
4. MSM model of inflation

The final objective of our analysis is an attempt to apply the Future Inflation Indicator in the inflation forecasting model. We expect that the FII embeds information (variables), that is leading with respect to the process of inflation, i.e. which can be used as a proxy for inflation expectations. At the beginning we decided to analyse separately the process of inflation and the process underlying the FII. The aim of such an analysis was to identify for both series independently the periods where their behaviour was relatively stable. Here by stable we mean that for such a period a model given by a linear regression would relatively well describe the data. Markov chains form a natural framework for this kind of analysis.

Our analysis comprises several steps. In the beginning we focus on each of the series separately to understand their dynamics. In the first stage a three state model MSI(3)-AR(2) with varying variation and autoregressive parameters was estimated.

\[
\text{fii}_t = \nu + \phi_{1,1} \text{fii}_{t-1} + \phi_{2,1} \text{fii}_{t-2} + \tilde{\nu}(s_i)
\]

where \(\text{fii}_t\) stands for the level of the FII in period \(t\) and \(\tilde{\nu}(s_i) \sim N(0, \sigma^2)\).

The results of the estimation show that the first state is persistent. This means that after the process leaves the state it never comes back to that state. The process entered the first state in August 1999 and never left it. That was the reason for cutting the period of analysis into the periods before and after 1999.

In order to detect different patterns of behaviour (different regimes) in the data generating process of inflation, at first inflation in Poland (\(\pi\)) was assessed with MSM based on the information from the whole sample (1992 – 2005). The first model selected with respect to the information criteria was MSI(2)-AR(2), which can be described by the following formula:

\[
\pi_t = \nu + \phi_{1,1} \pi_{t-1} + \phi_{2,1} \pi_{t-2} + \tilde{\nu}(s_i)
\]

Estimation performed on the whole sample shows that the hypothesis of a linear model can be rejected (using the LR test) in favour of a model with two states. The results show that the sample was divided into two sub-samples: state one – from the beginning of the sample until January 1995 and state two – from February 1995 onwards. In addition, state one proved to be connected with a significantly higher variance and the standard error was equal to 2.5, which is five times the magnitude of

59 MSI(n)-AR(m) refers to a model with two hidden states and m-th order autoregressive process of inflation in each state. If additionally Future Inflation Indicator is included it is referred as MSI(n)-ARX(m).
the standard error in state two. The interdependence is visible in figure 1, where the
time series before 1995 is much less stable.

Another important feature of state one is that it behaves almost as white noise and
due to this the shocks have a permanent effect. It confirms the hypothesis that before
1995 the inflation was not stationary and imposes to treat the period separately.

Both results of the estimations – for the FII and inflation – of data generating
processes led to a conclusion that both inflation and the FII where not stationary in the
early stages of the market economy in Poland. This might indicate that the relations
between the main macroeconomic variables (that are included in the FII) and inflation
were distorted in that period. Hence, we try to check whether the FII significantly
improves prediction of inflation just for the period after August 1999. Figure 3 depicts
the time series of the FII and inflation. In the chart one can notice that in both cases
when inflation increased, the turning points were led by upswings in the FII.

![Figure 3. Inflation (YoY) and Leading Indicator in the period between August 1999 and October 2005. Source: NBP and BIEC LCC.](image_url)
We estimate the following model:

$$\pi_t = \nu_t + \phi_{1,t} \pi_{t-1} + \phi_{2,t} \pi_{t-2} + \alpha_{3,t} \cdot \text{fii}_{t-3} + \alpha_{4,t} \cdot \text{fii}_{t-4} + \alpha_{5,t} \cdot \text{fii}_{t-6} + \eta_t(s)$$

(6)

Before we managed to obtain the final form of the model, various combinations of lagged values of the FII were considered. It appeared however that for only three lags (3M, 4M and 6M) estimates of the parameters are significant. Hence, it seems that the FII is a good leading indicator of inflation within a 3–6 months horizon. From Figure 3 it seems also that the FII is better in predicting incoming troughs than peaks of the inflation. It is also confirmed by the results of model estimation (see Tables 1-3). In state one, which corresponds to lower inflation, the value of t-Student statistics is the highest for \(\text{fii}_{t-6}\). In case of state two the best regressor appears to be \(\text{fii}_{t-3}\).

Table 1. Basic statistics for an MSI(2)-ARX(2) model for time series of inflation with FII.

<table>
<thead>
<tr>
<th>Sample:</th>
<th>August 1999</th>
<th>October 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Parameters</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>linear model</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>log-likelihood</td>
<td>-18.92</td>
<td>linear model: -38.49</td>
</tr>
<tr>
<td>AIC</td>
<td>0.93</td>
<td>linear model: 1.21</td>
</tr>
<tr>
<td>HQ</td>
<td>1.13</td>
<td>linear model: 1.3</td>
</tr>
<tr>
<td>SC</td>
<td>1.43</td>
<td>linear model: 1.43</td>
</tr>
<tr>
<td>linearity test LR</td>
<td>39.13</td>
<td>(\text{Chi}(7)=0.0000) ** (\text{Chi}(9)=0.0000) **</td>
</tr>
</tbody>
</table>

Source: Own computations
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Table 2. Characteristics of Switching Process for MSI(2)-ARX(2) model for time series of inflation with FII.

<table>
<thead>
<tr>
<th>Transition matrix:</th>
<th>state 1</th>
<th>state 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>state 1</td>
<td>0.86</td>
<td>0.14</td>
</tr>
<tr>
<td>state 2</td>
<td>0.21</td>
<td>0.79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>states:</th>
<th>observations</th>
<th>prob.</th>
<th>exp. length</th>
</tr>
</thead>
<tbody>
<tr>
<td>state 1</td>
<td>43.5</td>
<td>0.6</td>
<td>7.06</td>
</tr>
<tr>
<td>state 2</td>
<td>31.5</td>
<td>0.4</td>
<td>4.71</td>
</tr>
</tbody>
</table>

Source: Own computations

Table 3. Estimated parameters for MSI(2)-ARX(2) model for time series of inflation with FII.

<table>
<thead>
<tr>
<th>state 1</th>
<th>coeff.</th>
<th>std. error</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const(Reg.1)</td>
<td>-2.92</td>
<td>1.91</td>
<td>-1.52</td>
</tr>
<tr>
<td>PL1</td>
<td>0.94</td>
<td>0.09</td>
<td>10.96</td>
</tr>
<tr>
<td>PL2</td>
<td>-0.04</td>
<td>0.08</td>
<td>-0.51</td>
</tr>
<tr>
<td>FIL3</td>
<td>-0.11</td>
<td>0.07</td>
<td>-1.7</td>
</tr>
<tr>
<td>FIL4</td>
<td>0.28</td>
<td>0.09</td>
<td>3.04</td>
</tr>
<tr>
<td>FIL6</td>
<td>-0.13</td>
<td>0.04</td>
<td>-3.17</td>
</tr>
<tr>
<td>std. deviation:</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>state 2</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Const(Reg.2)</td>
<td>-28.09</td>
<td>10.82</td>
<td>-2.6</td>
</tr>
<tr>
<td>PL1</td>
<td>0.82</td>
<td>0.24</td>
<td>3.4</td>
</tr>
<tr>
<td>PL2</td>
<td>0.03</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td>FIL3</td>
<td>0.33</td>
<td>0.11</td>
<td>2.92</td>
</tr>
<tr>
<td>FIL4</td>
<td>-0.21</td>
<td>0.12</td>
<td>-1.79</td>
</tr>
<tr>
<td>FIL6</td>
<td>0.18</td>
<td>0.09</td>
<td>2.06</td>
</tr>
<tr>
<td>std. deviation:</td>
<td>0.28</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Own computations
An interesting conclusion can be drawn from the analysis of one period "forecast errors". Figure 4 shows that the model enhanced with the information from the FII improves forecasting features of the simple univariate model in terms of predicting the future hidden state. In times of regular inflation, forecast errors are quite similar in models with and without the FII. However, close to turning points, the advantage of the model with the FII is significant and, in particular, application of this model may help in avoiding mistakes around the turning points.

![Figure 4. Filtered residuals of the Markov switching model of inflation with FII and the residuals of its linear counterpart. Source: Own calculations.](image)

A better overview of the forecasting features of the model provides the statistical analysis of the forecast errors. In the case of the model with the FII the average error amounted to 0.0037 while the standard deviation amounted to 0.3547. The model without the FII was on average making larger forecasting mistakes and with higher deviation – the average was 0.0058 and the standard deviation 0.4233.

All in all, we show that inflation can be described using the MS models. We show that during the period of economic transition in Poland, which was associated with high level of inflation, the variation of inflation was not stable. This confirms the Ball-Friedman hypothesis for the case of Poland in early 1990s. Moreover, we find also a stable relation between the processes generating inflation and the FII in the period after
1999. In addition, inclusion of the information embedded in the FII, a proxy for inflation expectations, significantly improves the ability of the model to explain changes in the level of inflation, in particular around the turning points.

5. Conclusions

We show that Markov Switching models provide a suitable framework for modelling inflation in Poland. Using MS model framework we delimit the period of economic transition in the Polish economy, where the process of disinflation occurred. This delimitation was crucial, as in line with the Ball-Friedman hypothesis the periods of high inflation are associated with higher variance of inflation. We propose how to distinguish higher and lower inflation periods using the MS framework. Moreover, we show that MS models can be successful in modelling inflation in the low inflation era, i.e. in the period after 1999. Finally, we find that information embedded in the FII is very useful in modelling inflation process, as incorporation of the FII improves the forecasting power of models especially around the turning points. These conclusions should be verified for developed economies in order to establish whether inclusion of a leading indicator of similar construction can improve the quality of the models.
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